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# MAGNETIC MONOPOLES, GAUGE INVARIANT DYNAMICAL VARIABLES AND GEORGI GLASHOW MODEL[1]

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**ABSTRACT**

We investigate Georgi-Glashow model in terms of a set of explicitly SO(3) gauge invariant dynamical variables. In the new description a novel compact abelian gauge invariance emerges naturally. As a consequence magnetic monopoles occur as point like "defects" in space time. Their non-perturbative contribution to the partition function is explicitly included. This procedure corresponds to dynamical "abelian projection" without gauge fixing. In the Higgs phase the above abelian invariance is to be identified with electromagnetism. We also study the effect of $\theta$ term in the above abelian theory.

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## INTRODUCTION

The subject of magnetic monopoles has been fascinating ever since they were proposed by Dirac [1] in 1931 to explain electric charge quantization in abelian theory. As a consequence, the abelian gauge group becomes compact. It has been emphasized in the past [2, 3] that the global nature of the gauge group is important in determining the physical properties of a theory. In pure compact quantum electrodynamics (CQED) on lattice with angular gauge fields and therefore a compact U(1), the magnetic charges have dynamical origin and their condensation is responsible for the confining phase of this theory. Likewise, in non-abelian gauge theories with compact gauge groups, magnetic charges can occur naturally because of the hidden compact U(1) subgroup(s) [2, 3]. Their condensation could provide a sufficient framework to explain quark confinement along the lines of dual Meissner effect [4] in CQED. Based on the suggestion of 't Hooft [3], there have been many attempts in the past to get effective abelian theories with magnetic monopoles via "abelian projections". In these approaches certain collective excitations of the theory act as an effective "SU(2) adjoint Higg field" whose zeros are the possible locations of the magnetic charges. These issues have been largely addressed at the level of kinematics. However, unlike CQED, there has been very little progress at the level of dynamics besides Monte Carlo simulations [5]. The present work is an attempt to understand some of these issues along the lines of CQED. We will illustrate all the ideas with the simplest Georgi Glashow model (GGM) where the Higgs field is one of the microscopic fields present in the lagrangian itself. This model has been a crucial ground to develop some deep ideas related to non-perturbative dynamics in non-abelian gauge theories [6, 7, 2, 12]. In this paper using very simple idea of change of basis in the internal space of GGM, we show that a] It can be rewritten completely in terms of explicitly SO(3) gauge invariant fields, b] In terms of the above dynamical variables a novel *compact* abelian gauge invariance (*not a subgroup of SO(3)*) naturally emerges, c] As a consequence, the magnetic monopoles in the theory now are point like. We explicitly compute their contribution to the partition function. The results presented below are independent of the space time dimension but we illustrate the idea in d=4. The basic idea of this work is to define a co-moving orthonormal frame or "body fixed frame" ( BFF) in the internal space with one of its axes rigidly attached or identified with the unit Higgs isovector field $\hat{\phi}(x)$. Our motivation to use such frame

to describe the dynamics is the following: The other two axes of the BFF being arbitrary up to a local rotation around $\hat{\phi}(x)$, the dynamics in the BFF will have a built in abelian gauge invariance. Moreover, by expressing the angular motion of the above frame in terms of its "*angular velocities*" we will be able to construct certain SO(3) gauge covariant vector fields with some simple geometrical constraints. Therefore, the dynamics in terms of these covariant fields in the BFF will not only be SO(3) gauge invariant but will also have the much desired abelian gauge symmetry. The above description breaks down at the space time points ("defects") where $\vec{\phi}(x) = 0$ and will eventually lead to a non-perturbative description of magnetic charges. These BFFs have been used in the past [8] in the context of sigma model with global O(N) invariance. The GGM model is described by the lagrangian

$$\mathcal{L} = \frac{1}{2}D_\mu\vec{\phi}.D_\mu\vec{\phi} + \frac{1}{4}\vec{G}_{\mu\nu}(\vec{W}).\vec{G}_{\mu\nu}(\vec{W}) + V(|\vec{\phi}|). \tag{1}$$

Here $D_\mu\vec{\phi} \equiv \partial_\mu\vec{\phi} - e\vec{W}_\mu \times \vec{\phi}, \vec{G}_{\mu\nu} \equiv \partial_\mu\vec{W}_\nu - \partial_\mu\vec{W}_\nu - e\vec{W}_\mu \times \vec{W}_\nu$. The potential term $V(|\vec{\phi}(x)|)$ will not be crucial in the discussion below. In the case of pure gauge theories the above Higgs fields can be thought of as collective excitations of the gluonic fields present in the lagrangian. All the kinematical and most of the dynamical issues discussed below will remain unaltered. We describe the isovector Higgs by its gauge invariant magnitude and the direction, $\vec{\phi}(x) \equiv \rho(x)\hat{\phi}(x)$. The BFF is specified by three orthonormal vectors $\hat{\xi}^a(x)$ with $\hat{\xi}^3(x) \equiv \hat{\phi}(x)$. The dynamics of this frame can be described by a set of angular velocities defined by

$$D_\mu(\vec{\omega})\hat{\xi}(x) \equiv 0. \tag{2}$$

The "covariant derivative" $D_\mu^{ac}(\vec{\omega}) \equiv \delta^{ac}\partial_\mu - \epsilon^{abc}\omega_\mu^b$ is defined with respect to body fixed components of the angular velocities. Physically, it implies that any change in $\hat{\xi}^a(x)$, (a=1,2,3) is perpendicular to itself. We will denote the body fixed components of a vector $\vec{V}$ by latin indices and its components along $\hat{\xi}^\pm \equiv \hat{\xi}^1 \pm i\hat{\xi}^2$, by $V^\mp$. Defining the $U^{\hat{\phi}}(1)$ symmetry transformation as $\hat{\xi}^\pm(x) \rightarrow exp(\pm i\alpha(x))\hat{\xi}^\pm(x)$, one gets the following induced transformations on the angular velocities, $\omega_\mu^3(x) \rightarrow \omega_\mu^3(x) + \partial_\mu\alpha(x)$ and $\omega_\mu^\pm(x) \rightarrow exp(\pm i\alpha(x))\omega_\mu^\pm(x)$. Thus, under $U^{\hat{\phi}}(1)$ transformation $(\omega_\mu^\pm(x), \omega_\mu^3(x))$ transform like charged matter and abelian gauge field respectively. In the above

description in terms of angular velocities we have replaced the unit Higgs vector $\hat{\phi}(x)$ (i.e 2 angular degrees of freedom) by a set of 12 angular velocities. Therefore, this new dynamics is highly constrained. If we define a field strength tensor corresponding to matter fields $\hat{\phi}(x)$ by $\vec{F}_{\mu\nu}(\vec{\omega}(x)) \equiv \partial_\mu \vec{\omega}_\nu - \partial_\nu \vec{\omega}_\mu + \vec{\omega}_\mu \times \vec{\omega}_\nu$, then the constraints are simply $\vec{F}_{\mu\nu}(\vec{\omega}(x)) = 0$. In the space fixed frame (SFF) if $\theta(x)$ and $\psi(x)$ are the polar and azimuthal angles of $\hat{\phi}(x)$ then the solutions of the above equations are the "pure gauge" orthogonal matrices $O(\theta, \psi, \alpha)$ describing the relative orientation of the BFF and SFF in (2). We thus recover the $U^{\hat{\phi}}(1)$ gauge angle along with the 2 compact degrees of freedoms of $\hat{\phi}(x)$ which appeared explicitly in the partition function. Till now we have not described the hidden topological aspects of the theory. Nonabelian gauge theories also contain topological configurations with magnetic charge specified by their topology. Infact, t-Hooft Polyakov monopoles in Georgi Glashow model occur as stable solitons satisfying the classical equations of motions. In the context of abelian gauge theories with external magnetic charges or vortices [10], the multivalued angular nature of the matter fields plays crucial role in making topological properties manifest at the level of the partition function. This is equivalent to the role played by "compact photon" in CQED. Motivated by these abelian results, we characterise the BFF with Euler angular co-ordinates of $\vec{\phi}(x)$ with respect to the SFF, i.e, $\hat{\phi}(\vec{x}) = (cos\psi sin\theta,\ sin\psi sin\theta,\ cos\theta)$, $\hat{\xi}^1(\vec{x}) = (cos\psi cos\theta,\ sin\psi cos\theta,\ -sin\theta)$ and $\hat{\xi}^2(\vec{x}) = (-sin\psi,\ cos\psi,\ 0)$. The corresponding angular velocities in this frame are given by: $\omega^1_\mu = sin(\theta(x))\partial_\mu\psi(x)$, $\omega^2_\mu = -\partial_\mu\theta(x)$, $\omega^3_\mu = -cos(\theta(x))\partial_\mu\psi(x)$. We see that $\omega^3_\mu$ is not defined along the polar axis in the internal space where $\theta(x) = 0$ or $\pi$ and around which $\psi(x)$ can be multivalued. This is non-abelian analogue of decomposing complex abelian Higgs field into its radial and multivalued angular part. From now onwards the space time points where ($\hat{\phi} = (0, 0, \pm 1)$) with multivalued $\psi(x)$ will be called "singular points". Around these points the third component of the constraint $\vec{F}_{\mu\nu}(\vec{\omega}) = 0$ gets modifies by an extra contribution:

$$F^{3(np)}_{\mu\nu}(\vec{\omega}) = -cos\theta(x)(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\psi(x) \simeq \pm(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\psi^{sing}(x). \quad (3)$$

In the equation (3) the superscript $(np)$ stands for "non-perturbative" and its origin will become clear after we include the gauge fields. $\psi^{sing}(x)$ is the multivalued part of $\psi(x)$. Uptill now the above results were only regarding the kinematical aspects of the Higgs fields. We now extend these

kinematical aspects to include the gauge fields. The induced $U^{\hat{\phi}}(1)$ transformations on the body fixed components of the gauge fields are, $W^3_\mu(x) \rightarrow W^3_\mu(x), W^\pm_\mu(x) \rightarrow exp(\pm i\alpha(x))W^\pm_\mu(x)$. Under SO(3) gauge transformation $(\vec{\lambda})$ the BFF undergoes a rotation $\hat{\xi}^a \rightarrow \hat{\xi}^a + \vec{\lambda} \times \hat{\xi}^a$. Therefore, the angular velocities (2) transform as $\vec{\omega}_\mu \rightarrow \vec{\omega}_\mu + \vec{\lambda} \times \vec{\omega}_\mu - \partial_\mu \vec{\lambda}$. At this stage we can straight away define SO(3) covariant gauge fields $\vec{Z}_\mu(x)$ and their $U^{\hat{\phi}}(1)$ transformation laws by:

$$\begin{aligned} \vec{Z}_\mu(x) &\equiv e\left(\vec{\omega}_\mu(x) + e\vec{W}_\mu(x)\right) \\ Z^\pm_\mu(x) \rightarrow exp(\pm i\alpha(x))Z^\pm_\mu(x), &\quad Z^3_\mu(x) \rightarrow Z^3_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x). \end{aligned} \tag{4}$$

From now on we will often denote $Z^3_\mu$ by $A_\mu$. Under SO(3) gauge transformations the $\vec{Z}_\mu$ transforms like a vector and therefore its components in the BFF are explicitly gauge invariant. This was one of the motivations to describe the dynamics in the moving frame. With slight abuse of language we will call $(A_\mu(x), Z^\pm_\mu(x))$ as the "photon" and the corresponding charged matter fields. We will now proceed to define the quantum field theory in terms of these SO(3) gauge invariant fields with $U^{\hat{\phi}}(1)$ gauge invariance. Using the constraints (3) and $F^\pm_{\mu\nu}(\omega(x)) \simeq 0$, we find that

$$\vec{G}_{\mu\nu}(\vec{W}) = \left(\partial_\mu Z^a_\nu(x) - \partial_\nu Z^a_\mu(x) - e\epsilon^{abc}Z^b_\mu(x)Z^c_\nu(x)\right)\hat{\xi}^a(x) + \tfrac{1}{e}\vec{F}^{(np)}_{\mu\nu}(\psi^{sing}).$$

$$D_\mu(\vec{W})\hat{\phi}(x) \quad = \quad \epsilon^{\alpha\beta}Z^\alpha_\mu(x)\hat{\xi}^\beta(x) \tag{5}$$

Here $(\alpha, \beta)$ vary from 1 to 2. At regular points with $\vec{F}^{(np)}(x) = 0$ the equations (5) are easy to interpret. Under the SO(3) gauge transformation both sides of these equations transform covariantly and $\hat{\xi}^a(x)$ itself being covariant, $Z^a_\mu(x)$ (a=1,2,3) are left invariant. At singular points or equivalently the non-perturbative sector of the theory, only the third component of $\vec{F}^{(np)}_{\mu\nu}(\psi^{sing})$ along the $\hat{\phi}$ direction contributes to the flux with strength proportional to $(\frac{1}{e})$. This singular flux is given by $\pm\frac{2\pi N}{e}$ where $N$ is the homotopy index of the mapping $S^1_{physical} \rightarrow S^1_{internal}$ provided by the multivalued $\psi^{sing}$ field. The Noether current of the symmetry transformation (4) is $J^{\hat{\phi}}_\nu = 2eIm\Big(Z^-_\mu D_\mu(A)Z^+_\nu + Z^+_\mu D_\nu(A)Z^-_\mu - \frac{1}{2}\partial_\mu\left(Z^+_\mu Z^-_\nu\right)\Big)$. The $U^{\hat{\phi}}(1)$

covariant derivative $D_\mu(A)$ is defined as $D_\mu(A(x))Z^{\pm}_\nu \equiv \partial_\mu Z^{\pm}_\nu \mp ieA_\mu Z^{\pm}_\nu$. $J^{\hat\phi}_\nu$ is explicitly SO(3) invariant and is independent of the initial SO(3) Noether currents. Therefore, the abelian field strength tensor and the corresponding Maxwells equations are given by

$$\begin{aligned} F^{U(1)}_{\mu\nu} &= \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - \frac{1}{e}F^{3(np)}_{\mu\nu} \\ \partial_\mu F^{U(1)}_{\mu\nu} &= J^{\hat\phi}_\nu \quad , \qquad \partial_\mu \tilde F^{U(1)}_{\mu\nu} \equiv K^{mag}_\nu. \end{aligned} \tag{6}$$

Here $\tilde F_{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\tilde F_{\rho\sigma}$. The last equation in (6) is the Bianchi identity. The magnetic current $K^{mag}_\nu = \frac{1}{e}\partial_\nu \tilde F^{3(np)}_{\mu\nu}$ is the topological current which is trivially conserved. Note that it is nonzero only at "defects" where $\vec\phi(x) = 0$ and is also SO(3) gauge invariant. The above singular points where $F^{3(np)}_{\mu\nu}$ is non zero form a 2 dimensional manifold (Dirac world sheet) which can be characterised by two parameters $(\sigma_1, \sigma_2)$ and a point on this sheet is given by a four vector $X_\mu(\sigma_1, \sigma_2)$. We will also choose $\sigma_1$ to describe the time evolution. In the above abelian gauge theory $F^{3(np)}_{\mu\nu}$ should be thought of as *multivalued* $U^{\hat\phi}(1)$ gauge transformation on $A_\mu$ by $-\psi^{sing}(\theta = 0)$ and $+\psi^{sing}(\theta = \pi)$ along the trajectories $X_\mu(\sigma_1, \sigma_2)$ at fixed time. Therefore, these correspond to two infinitely thin Dirac string with $\pm(\frac{2\pi N}{e})$ unit of magnetic flux respectively. The magnetic monopoles (anti-monopoles) are located at points where these two strings meet or $\hat\phi$ flips between (0,0,+1) and (0,0,−1). It is clear that the origin of the above Dirac strings is the same as that of the Abrikosov Nielsen Olesen vortices in the abelian Higgs model. Moreover, the flipping of the extra (*non-abelian*) degree of freedom of the Higgs field described by its polar angle $\theta(x)$ in its present angular description provides the above strings with "defects" which are the gauge invariant locations of the magnetic charges in the theory. Infact, this construction of monopoles in non-abelian gauge theory is similar to that of in CQED. The $(F^{U(1)}_{\mu\nu})^2$ part of the action defined in (6) is just the naive continuum limit of the pure CQED action. In the following we will extend the above analogies further. To recast the magnetic term in a more standard form in the partition function, we divide the measure over azimuthal angle into singular and regular parts [10] at each space time point i.e $\int d\psi(x) = \int d\psi^{reg}(x) \int d\psi^{sing}(x)$, here $\psi^{sing}$ is the multivalued part contributing to the right hand side of (3). Noting that the integrations over $\hat\phi(x)$ can be replaced by the Haar measure over the orthogonal matrices

$O(\theta(x), \psi(x), \alpha(x))$, the single valued angular integration can be traded off with the integration over the angular velocities by the following two identities:

$$\int d\vec{\omega}^a_\mu(x)\delta\left(\vec{\omega}^a_\mu(x) - Tr\left(L^{(a)}O(x)\partial_\mu O^{-1}(x)\right)\right) \equiv 1$$
$$\int dO\,(\theta,\psi,\alpha)\,\delta(\omega^a_\mu - Tr\,(L^a O\partial_\mu O^{-1})\,) = \int d\psi^{sing}\delta\left(F^a_{\mu\nu}\,(\vec{\omega}) + F^{a(np)}_{\mu\nu}(\psi^{sing})\right)$$

We have suppressed the product over space time points, Lorentz and color indeces in the above measures. The action has only implicit dependence on $\omega_\mu$ through $Z_\mu$. Therefore, changing the variables in the measure from $\vec{W}_\mu(x)$ to $\vec{Z}_\mu(x)$, we get the partition function only in terms of the gauge invariant fields $(\rho(x), \vec{Z}_\mu(x))$ and the term containing the topological magnetic currents of the theory. We will now rewrite this term with Dirac strings. It is convinient to describe the above singularities in a generic configuration in terms of gauge invariant world lines of the monopoles $X^i(\sigma_1)$ and a set of corresponding integers $m^i$ describing their magnetic charges in the units of $\frac{4\pi}{e}$. A generic monopole current is given by $K_\mu(x) = \sum_{i=1}^{\infty} m^i \int d\sigma_1 \frac{dX^i_\mu(\sigma_1)}{d\sigma_1}\delta^4(x - X^i(\sigma_1))$ Therefore, the magnetic term in (6) is given by

$$\tilde{F}^{3(np)}_{\mu\nu}(x) = \pm\frac{4\pi}{e}\sum_{i=1}^{\infty} m^i \int d^2\sigma\epsilon^{\alpha\beta}\left(\partial_\alpha X^i_\mu(\sigma)\partial_\beta X^i_\nu(\sigma)\right)\delta^4\left(x - X^i(\vec{\sigma})\right). \quad (7)$$

This is just the term introduced by Dirac [9] in the context of abelian gauge theory where point particles carrying magnetic charges were put by hand. The partition function corresponding to (1) now is given by

$$\begin{aligned} Z &= \sum_{m_1,..,m_\infty}\prod_{i=1}^{\infty}\int dX^i_\mu(\vec{\sigma})J(X_\mu)\int\rho^2 d\rho\int dZ^a_\mu exp - S\left(\rho, \vec{Z}_\mu, X^i_\mu\right) \\ S &= \int\Big[\frac{1}{4}\left(F^{U(1)}_{\mu\nu}\right)^2 + \frac{1}{4}\left(D_\mu(A)Z^+_\nu - D_\nu(A)Z^+_\mu\right).h.c + \frac{ie}{2}F^{U(1)}_{\mu\nu}Z^+_\mu Z^-_\nu \\ &- \frac{1}{16}\left(Z^+_\mu Z^-_\nu - h.c\right)^2 + \frac{1}{2}\left(e^2\rho^2 Z^+_\mu Z^-_\mu + (\partial_\mu\rho)^2\right) + V(\rho)\Big]d^4x \qquad (8) \end{aligned}$$

Here J(X) is the Jacobian [11] due to the change of the measure to the string world sheet. Note that the $U^{\hat{\phi}}(1)$ invariance (4) of the partition function (8)

is manifest. This is an exact result and no gauge fixing has been done. In the broken phase where $\rho(x)$ = constant + fluctuations, the above partition function describes the interaction of photon with the charged massive spin 1 gauge bosons and magnetic monopoles. The physical fields here are explicitly gauge invariant and have the right electric charges under $U^{\hat{\phi}}(1)$. It is easy to see that by choosing the unitary gauge $\hat{\phi}(x) = (0,0,1)\forall x$, possible only in the perturbative sector $(m^i = 0, \forall i)$, we recover the standard results. At this stage, having an exact abelian theory with magnetic monopoles in the partition function, we can also convert them into dyons by adding the CP violating $\theta$ term [12] in the action (1): $\Delta\mathcal{L} = \theta\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^{U(1)}_{\mu\nu}(A_\mu)F^{U(1)}_{\rho\sigma}(A_\mu)$ The equation of motion of $A_\mu$ in (6) are now modified and acquires a $\theta$ dependent term: $\partial_\mu F^{U(1)}_{\mu\nu} = J^{\hat{\phi}}_\nu + \frac{\theta e^2}{8\pi^2}K^{mag}_\nu$. Therefore, all the magnetic charges with strengths $(m_1, .....m_\infty)$ also acquire electric charges $(q_1, .....q_\infty) = \frac{e\theta}{2\pi}(m_1, .....m_\infty)$ leading to generalized Schwinger quantization condition. The term added above is not a surface term because of the Dirac strings. It is also interesting to contrast the above formulation with some of the standard results in the literature. In the broken phase and in the Higgs vacuum: $D_\mu(\vec{W})\hat{\phi}(x) \approx 0$. Therefore, $e\vec{W}_\mu(x) \approx \hat{\phi}(x) \times \partial_\mu\hat{\phi}(x) + e\hat{\phi}(x)\tilde{A}_\mu(x)$ and $\tilde{A}_\mu(x) \equiv \hat{\phi}(x).\vec{W}_\mu(x)$ is identified with the photon field. Our identification of photon differs from this by the third component of the angular velocity and is explicitly gauge invariant. The abelian field strength tensor defined in (6) is the same as the one proposed in [6]. The other proposal in the literature is simply $\hat{\phi}(x).\vec{G}_{\mu\nu}(x)$. However, this will not correspond to (6).

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